Conformal Mapping Exercises

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Additional Resources:

1. *Complex analysis* by Gamelin
2. `https://math.mit.edu/~jorloff/18.04/notes/topic10.pdf`

Problems

1. Map the upper half disk $\{z\colon |z| < 1, 0 < \arg(z) < \pi\}$ to the upper half plane using linear fractional transformations

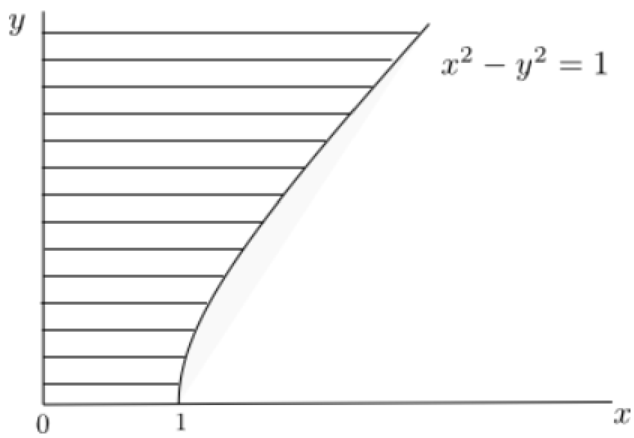
2. (2007 September #4) Map the upper half plane $y > 0$ of the z-plane conformally onto the semi-infinite strip $u > 0$, $-\pi < v < \pi$ in the w-plane

3. (2005 September #5) Construct a one-to-one conformal mapping of the region which is the exterior of the two circles $|z + \pm 1| = 1$ onto the disk $|w| < 1$, and such that $z = \infty$ is mapped to $w = 0$.
$$R = \{z\colon |z - 1| > 1, |z + 1| > 1\}$$
You may represent the mapping as a composition of a number of simple maps, each one of which should be written down explicitly. You need not write the overall map explicitly.

   (Hint: It may be useful to start by sending to $z = 0$ to ∞)

4. (2004 January # 4)

   The shaded region seen in the picture is to be mapped one-to-one onto the upper half-plane so that $0, 1, \infty$ be mapped to $0, 1, \infty$



5. Map the slit disk $\mathbb{D} - [1/2, 1)$ to the unit disk $\mathbb{D}$.

6. Let D_1, D_2 be two open simply connected sets not equal to $\mathbb{C}$. Describe all conformal maps between D_1 and D_2

7. Map the slit strip $\{z\colon\ \mathrm{Im}(z) \leq \pi\} \setminus \{\pi/2i + t\colon t \geq 0\}$ to the strip $\{z\colon 0\,\mathrm{Im}(z) \leq \pi\}$

8. (2016 September #1) Find a conformal map between the following domains:

   (a) from $\mathbb{R} \times (0, \pi)$ to $\mathbb{H} = \{z, \mathrm{Im}(z) > 0\}$

   (b) From the disk $\mathbb{D} = \{z\colon |z| < 1\}$ to $\mathbb{H}$

   (c) from $\mathbb{H} \setminus [0, ir]$ to $\mathbb{H}$, where $r > 0$